

## COMPARISON OF VARIOUS OPTIMIZATION TECHNIQUES FOR DESIGN FIR DIGITAL FILTERS

Amanjeet Panghal<sup>1</sup>, Nitin Mittal<sup>2</sup>, Devender Pal Singh<sup>3</sup>, R.S. Chauhan<sup>4</sup>, Sandeep K. Arya<sup>5</sup>.

<sup>1,2</sup> Electronics & Comm. Deptt., MIET, Mohri

<sup>3</sup> Central Scientific Instruments Org., Chandigarh

<sup>4</sup> Electronics & Comm. Deptt., JMIT, Radaur.

<sup>5</sup> Electronics & Comm. Deptt., GJUS&T, Hisar.

eced\_miet@yahoo.com

**Abstract-** In this paper different method for design of FIR Filters with low group delay is proposed. The design method is based on Parks McClellan algorithm and Sequential Optimization. In sequential optimization, Newton and Quasi-Newton method is used. In this paper we compare the various optimization techniques and the sequential optimization performs better in comparison to Parks McClellan algorithm. The reason to formulate and solve the design problem in a sequential optimization formulation is that the complementarily conditions associated with the sequential optimization lead to a very small number of non-zero multipliers that need to be updated in a given iteration. This in turn improves design efficiency as well as the algorithm's numerical stability which is of critical importance for the design filter. Design examples with comparisons are presented to illustrate the effectiveness of the sequential optimization method.

**Keywords:** FIR, Parks McClellan Algorithm, Newton Method, Sequential Optimization.

### 1. INTRODUCTION

Over the past several decades the field of Digital Signal Processing (DSP) has grown to important both theoretically and technologically. In DSP, there is two important types of Systems. The first type of systems performs signal filtering in time domain and hence it is known as DIGITAL FILTERS. The second type of systems provide signal representation frequency domain and are known as Spectrum Analyzer. Digital filtering is one of the most powerful tools of DSP. Digital filters are capable of performance specifications that would, at best, be extremely difficult, if not impossible, to achieve with an analog implementation. In addition, the characteristics of a digital filter can be easily changed under software control. Digital filters[1,2] are classified either as Finite duration unit pulse response (FIR) filters or Infinite duration unit pulse response (IIR) filters, depending on the form of unit pulse response of the system. In the FIR system, the impulse response sequence is of finite duration, i.e., it has a finite number of non zero terms.

Digital filters are classified as Recursive and Non-Recursive filters. The response of Recursive or FIR filters depend only upon Present and previous input of signal. FIR filters have the following advantages:-

- They can have an exact linear phase.
- They are always stable.
- The design methods are generally linear.
- They can be realized efficiently in hardware.
- The filter start up transients has finite duration.

Digital filters are integral parts of many digital signal processing systems, including control systems,

systems for audio and video processing, communication systems and systems for medical applications. Due to the increasing number of applications involving digital signal processing and digital filtering the variety of requirements that have to be met by digital filters has increased as well. Consequently, there is a need for flexible techniques that can design digital filters satisfying sophisticated specifications. This paper presents methods for the sequential optimization design of digital filters.

The Parks-McClellan algorithm and its variant have been most efficient tools for the minimax design of FIR digital filters [2]. However, these algorithms apply only to the class of linear-phase FIR filters the group delay introduce by these filters is constant and independent of frequency in the entire baseband but it can be quite large. In practice, a variable group delay in stopband is of little concern and by allowing the phase response to be nonlinear in stopbands, FIR filter can be designed with constant group delay with respect to the passbands which is significantly reduced relative to that achieved with filters that have a constant group delay the entire baseband.

This paper presents a least-p<sup>th</sup> approach to the design of low delay FIR filters. For FIR filter, the weighted  $L_p$  error function with an even integer  $p$  can be shown to be globally convex. This property, in conjunction with the availability of the gradient and hessian of the objective function in closed form, enable us to develop an optimization method for the design of FIR filter.

## 2. PROBLEM FORMULATION

Consider a FIR digital filter with transfer function

$$H(z) = \sum_{n=0}^N h_n z^{-n} \quad (1)$$

Such that the weighted  $L_{2p}$  approximation error

$$f(h) = \left[ \int_0^\pi W(\omega) |H(e^{j\omega}) - H_d(\omega)|^{2p} d\omega \right]^{\frac{1}{2p}} \quad (2)$$

is minimized, where  $W(\omega) \geq 0$  is weighting function is a positive integer, and

$$h = [h_0 h_1 \dots \dots \dots h_N]^T$$

if it is assumed.

$$H_d(\omega) = H_{dr}(\omega) - jH_{di}(\omega)$$

$$c(\omega) = [1 \cos \omega \dots \dots \cos N\omega]^T$$

$$s(\omega) = [0 \sin \omega \dots \dots \sin N\omega]^T$$

Then Eq (2) become

$$f(h) = \left\{ \int_0^\pi W \left[ (h^T c - H_{dr})^2 + (h^T s - H_{di})^2 \right]^p d\omega \right\}^{1/2p} \quad (3)$$

Where for simplicity the frequency depends of  $W$ ,  $c$ ,  $s$ ,  $H_{dr}$  and  $H_{di}$  has been omitted. Now if it is consider that

$$e_2(\omega) = \left[ h^T c(\omega) - H_{dr}(\omega) \right]^2 + \left[ h^T s(\omega) - H_{di}(\omega) \right]^2 \quad (4)$$

then the objective function can be expressed

$$f(h) = \left[ \int_0^\pi W(\omega) e_2^p(\omega) d\omega \right]^{1/2p} \quad (5)$$

Using Eq. (5), the gradient and hessian of objective  $f(h)$  function can be readily obtained as

$$\nabla f(h) = f^{1-2p}(h) \int_0^\pi W(\omega) e_2^{p-1}(\omega) q(\omega) d\omega \quad (6a)$$

where

$$q(\omega) = \left[ h^T c(\omega) - H_{dr}(\omega) \right] c(\omega) + \left[ h^T s(\omega) - H_{di}(\omega) \right] s(\omega) \quad (6b)$$

and

$$\nabla^2 f(h) = H_1 + H_2 - H_3 \quad (6c)$$

where

$$H_1 = 2(p-1) f^{1-2p}(h) \int_0^\pi W(\omega) e_2^{p-2}(\omega) q(\omega) q^T(\omega) d\omega \quad (6d)$$

$$H_2 = f^{1-2p}(h) \int_0^\pi W(\omega) e_2^{p-1}(\omega) \left[ c(\omega) c^T(\omega) + s(\omega) s^T(\omega) \right] d\omega \quad (6e)$$

$$H_3 = (2p-1) f^{-1}(h) \nabla f(h) \nabla^T f(h) \quad (6f)$$

respectively. Of central importance to the present algorithm is the property that for each and every positive integer  $p$ , the weighted  $L_{2p}$  objective function defined in Eq.(2) is convex in the entire parameter space  $R^{N+1}$ . This property can be proved by

showing that the Hessian  $\nabla^2 f(h)$  is positive semi definite for all  $h \in R^{N+1}$ .

## 3. DESIGN ALGORITHMS

An FIR filter whose frequency response approximates a rather arbitrary frequency response  $H_d(\omega)$  to within a given tolerance in the minimax sense can be obtained by minimizing  $f(h)$  in Eq (2).with a sufficient large  $p$ . For a given  $p$ ,  $f(h)$  has a unique global minimizer. Therefore, any descent minimization algorithm e.g. Newton and quasi Newton can be used to obtained minimax. The amount computation required to obtain the design is largely determined by the choice of optimization method as well as the initial point assumed.

A reasonable initial point can be deduced by using the  $L_2$ -optimal design obtained by minimizing  $f(h)$  in Eqn (2) with  $p=1$ .It can be written as

$$f(h) = (h^T Q h - 2h^T \rho + k)^{1/2} \quad (7a)$$

$$Q = \int_0^\pi W(\omega) \left[ e(\omega) e^T(\omega) + s(\omega) s^T(\omega) \right] d\omega \quad (7b)$$

$$P = \int_0^\pi W(\omega) \left[ H_{dr}(\omega) e(\omega) + H_{di}(\omega) s(\omega) \right] d\omega \quad (7c)$$

Since  $Q$  is positive definite, the global minimizer of  $f(h)$  in Eqn (7a) can be obtained as the solution of the linear equation

$$Qh = p \quad (8)$$

Since  $Q$  in Eq (8) is a symmetric Toeplitz matrix [4] for which fast algorithms are available to compute the solution of 8[6].

The minimization of convex objective function  $f(h)$  can be accomplished in a number of ways. Since the gradient and Hessian of  $f(h)$  are available in closed-form and  $\nabla^2 f(h)$  is positive semidefinite, the Newton method and the family of Quasi-Newton methods are among the most appropriate.

From Eq(5) and (6), we note that  $f(h)$ ,  $\nabla f(h)$ , and  $\nabla^2 f(h)$  all involve integration which can be carried out using numerical methods. In computing  $\nabla^2 f(h)$ , the error introduced in the numerical integration can cause the hessian to lose its positive definiteness but the problem can be easily fixed by modifying  $\nabla^2 f(h)$  to  $\nabla^2 f(h) + \varepsilon I$  where  $\varepsilon$  is a small positive scalar.

## 4. DIRECT AND SEQUENTIAL OPTIMIZATION

With a power  $p$ , weighting function  $W(\omega)$ , and an initial  $h$ , chosen, the design can be obtained directly or indirectly.

In direct optimization, one of the unconstrained optimization methods is applied to minimize the  $L_{2p}$  objective function in Eq.(5) directly. Based on rather

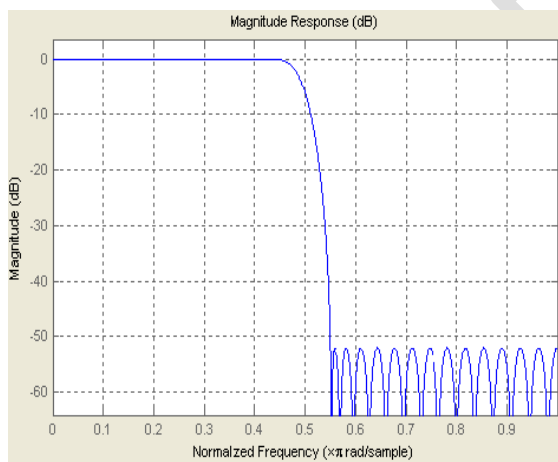
extensive trials, it was found that to achieve a near – order FIR filters a value comparable to the filter order N should be used.

In sequential optimization, an  $L_{2P}$  optimization is first carried out with  $p=1$ .the minimizer thus obtained, h is then used as the initial point in another optimization with  $p=2$ .the same procedure is repeated for  $p= 4, 8, 16...$  until the reduction in the objective function between two successive optimization is less than a prescribed tolerance.

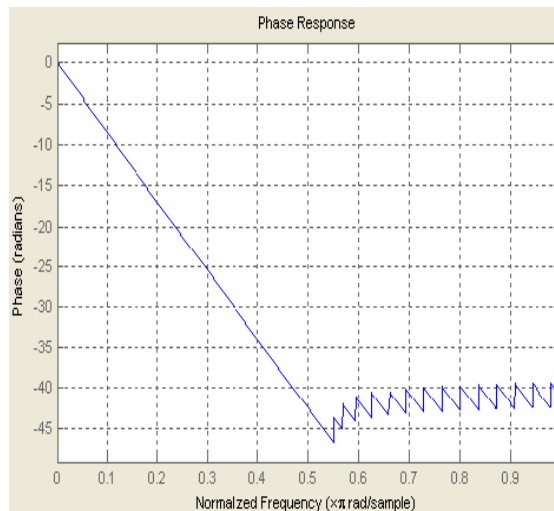
### 5. DESIGN EXAMPLES

The method applied through MATLAB is applied to design a low pass FIR filters with low pass band group delay.

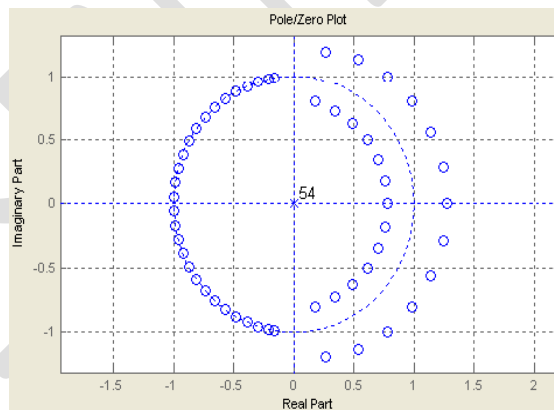
**Example 1**-Design a low pass FIR filter of order  $N=45$  that would have approximately constant pass band group delay of 23s. Assume idealized pass band and stop band gains of 1 and 0, respectively : normalized sampling frequency  $\omega_a = \pi$ ; passband edge  $\omega_p = 0.5\pi$ ; and stop band edge  $\omega_s = 0.54\pi$ ;  $W(\omega)=1$  in both the pass band and stop band ,and  $W(\omega)=0$  elsewhere. The magnitude response, pole zero response of FIR filter using the Parks-McClellan algorithm is depicted in figure 1a , b and c respectively.



(a)



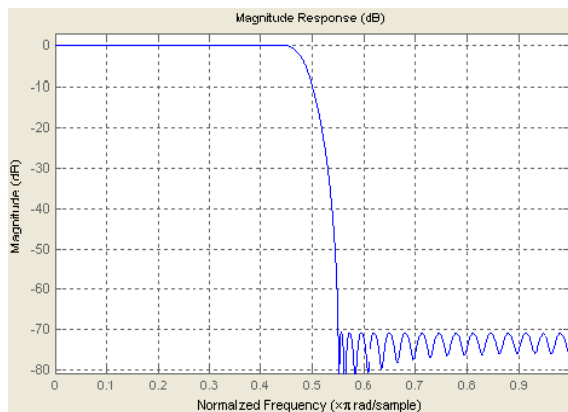
(b)



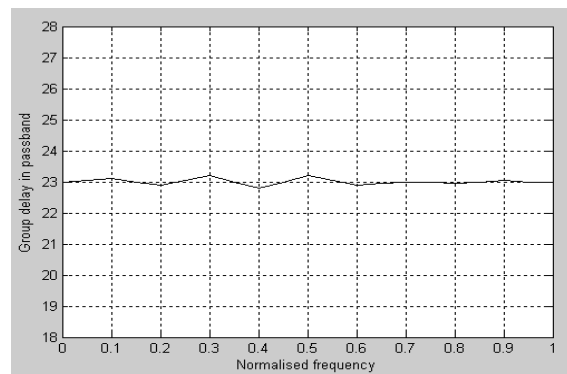
(c)

Figure 1:-Design of low pass FIR filter using Parks McClellan Algorithm for example 1 (a) Magnitude response (b) phase response (c) pole zero plot.

The magnitude response, phase response , pole zero plot and group delay in pass band of FIR filter for example 1 using sequential optimization are depicted in figure 2 a, b c and d respectively. It is interesting to note that the equiripple amplitude response is achieved in both pass band and stop band. The pass band group delay varies from 22.9 and 23.1 but it is not equiripple. This is because the sequential optimization is carried out for the complex-valued frequency response, not the phase response alone.

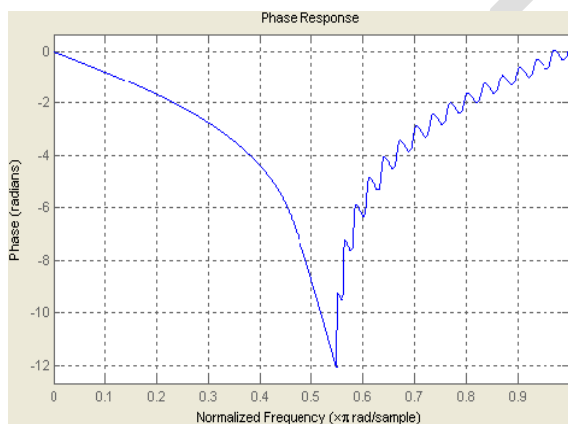


(a)

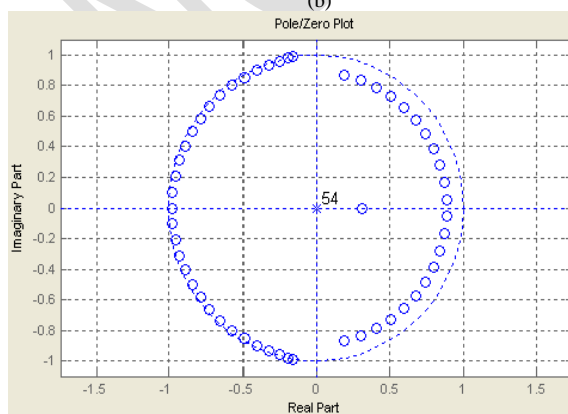


(d)

Figure 2:-Design of FIR filter with low pass group delay using Sequential optimization for example 1(a) magnitude response (b) phase response (c) pole zero plot (d) group delay in pass band



(b)



(c)

The design of FIR filter in figure 1a, b and c is implemented using traditional method of optimization (Parks-McClellan algorithm), while in figure 2 a,b,c and d we discuss the implementation of sequential optimization. In this paper from fig 1(c) zeroes lies outside the circle therefore the system is less stable in comparison pole zero plot in fig 2 (c). Therefore the stability in sequential optimization technique is much higher than in the traditional method of optimization (Parks-McClellan algorithm).

## 6. CONCLUSION

This paper present various optimization techniques for design of low pass FIR digital filters. In this paper we design low pass fir digital filter with Parks-McClellan algorithm and sequential optimization. The Parks-McClellan algorithm and its variant have been most efficient tools for the minimax design of FIR digital filters. However, these algorithms apply only to the class of linear-phase FIR filters the group delay introduce by these filters is constant and independent of frequency in the entire base band but it can be quite large, but there is a requirement of relatively non-linear FIR filter. Design examples presented in the paper have indicated that the method can be used to design relatively higher order and nonlinear phase FIR filter that are optimal in the minimax sense. In this paper from pole zero plots that the stability in sequential optimization technique is much higher than in the traditional method of optimization (Parks-McClellan algorithm).

## 7. REFERENCES

- [1] T. W. Parks and J. H. McClellan, Chebyshev approximation for nonrecursive digital filters with linear phase,"IEEE Trans.Circuit Theory, vol19, pp, 189-194, 1972.
- [2] A. Antoniou, Digital Filters: Analysis, Design and Applications, 2<sup>nd</sup> ed.McGraw Hill, 1993.

[3] Shpak, D.J. and A. Antoniou, "A generalized Remez method for the design of FIR digital filters," *IEEE Trans. Circuits and Systems*, pp. 161-174, Feb. 1990.

[4] I. W. Selesnick and C. S. Burrus, "Exchange algorithm that complement the Parks-McClellan algorithm for linear-phase FIR filter design," *IEEE Trans. on Circuits and Systems*

[5] J. J. Craig, Introduction to robotics, 2nd ed. Addison-Wesley, 1989.

[6] M. Werman and D. Weinshall, Similarity and affine invariant distance between 2D point sets", *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol, 17, pp.810-814, August 1995.

[7] I. W. Selesnick and S. Burrus, "Maximally flat low-pass FIR filters with reduced delay," *IEEE Trans. Circuits Syst. II*, vol. 45, no. 1, pp. 53--68, 1998.

[8] Nagel, P., Frequency-domain design method for linear-phase and minimum-phase FIR equalizers. *IEEE Trans. Circuits Syst.* v47 i11. 1347-1351.

[9] R.Fletcher, Practical Methods of Optimization, 2<sup>nd</sup> ed., Wiley, 1987.

[10] Rabiner, L.R., J.H. McClellan, and T.W. Parks, "FIR Digital Filter Design Techniques Using Weighted Chebyshev Approximations," *Proc. IEEE* 63 (1975).

[11] O. Hermann, and H. W. Schüssler, "Design of nonrecursive digital filters with minimum phase," *Electron. Lett.* 6, pp. 329-330, 1970.

[12] J. H. McClellan, T. W. Parks, and L. R. Rabiner, "A computer program for designing optimum FIR linear phase digital filters," *IEEE Trans. Audio Electroacoust.* AU-21, 506-526, 1973.

[13] I. W. Selesnick and C. S. Burrus, "Exchange algorithms that complement the Parks-McClellan algorithm for linear-phase FIR filter design," *IEEE Trans. on Circuits and Systems II*, 44(2):137-142, February 1997.

[14] O. Hermann, and H. W. Schüssler, "Design of nonrecursive digital filters with minimum phase," *Electron. Lett.* 6, pp. 329-330, 1970.

[15] Lightner, M.R.; Erdogmus, D.; 'Signal processing challenges in cognitive assistive technology [Exploratory DSP] ', *Signal Processing Magazine, IEEE* Volume 25, Issue 5, September 2008 Page(s):103 – 108

[16] B. Dumitrescu, R. Bregovic, T. Saramäki - Design of Low-Delay Nonuniform Oversampled Filter banks, *Signal Processing*, vol.88, no.10, pp.2518-2525, Oct. 2008.