Analytical approach of using the squeezed state formation of light for conducting all-optical noise free XOR logic operation.

Shyamal Kumar Pal* and Sourangshu Mukhopadhyay
Department of Physics, The University of Burdwan, Golapbag-713104, West Bengal, India
*Corresponding author.
Skpal5@gmail.com, sourangshu2004@gmail.com

Abstract:
Due to highly noise-reduced signal nature the squeezed state of light has drawn an enormous attention of present day scientific community specially in the area of information processing and digital communication. On the other side optics has been established as a powerful candidate to be used in information processing due to its high parallelism and real time speed of operation. Here in this paper the authors propose a new concept of developing all-optical XOR logic gate (which is a fundamental and essential one in digital communication and computation) using squeezed state formation of light.

1. Introduction:
Four-wave mixing and self-phase modulation have drawn the considerable attention since the pioneering work with the use of CW [1] and pulsed light [2] for the generation of squeezed state of light in optical fiber. Several new schemes to generate squeezed light by exploiting the Kerr nonlinearity in optical fibers have been demonstrated [3,4,5,6]. Parametric amplification, degenerate and non-degenerate four wave mixing, fibre optic ring interferometric processes, etc. are the optical nonlinear processes which are commonly responsible for the generation of squeezed state of light in different cases. Squeezed states are concerned with the minimum uncertainty states, when the classical monochromatic EM waves are compared with the quantum mechanical coherent states. In general a squeezed state has less noise in one quadrature than that of a coherent state.

Again switching is the inherent and essential parts in all optical distant communication and digital information processing. For optical networks and superfast information highways ultra speed signal processing system is essential. In case of optoelectronic systems signal processing speed is restricted due to optical to electrical and electrical to optical conversions. So an all-optical can’t only satisfy the requirements of ultra high-speed processing. To support the ultra high speed in future optical networks and to avoid the bulk optical-electrical-optical conversion based time delay, all optical switching are being more and more promising as essential components.

Interestingly in last few decades several all optical logic works are reported [7,8,9,10]. At the same time several all-optical logic based devices have also been implemented physically. In several applications of quantum communication, both theoretical as well as experimental investigations are strongly found on quantum states. As far our knowledge goes none of the works utilized squeezed state of light to implement optical logic or optical logic based digital systems. Therefore to support a meaningful realization of using squeezed state of light in logic operation the authors present here a new concept of all optical switching using squeezed state of light. Basically the non-linear Kerr effect is capable for the formation of a squeezed state. It produces an intensity dependence refractive index that can motivate an initial symmetric phase space distribution into an ellipsoidal shape. To overcome the “electronic bottleneck”, in electronic signal processing Semiconductor Optical Amplifier (SOA) based all optical switching is found then very much useful in all optical networks. Many all-optical/photonics devices have been reported based on SOA where the SOA have been used mainly. Cross gain modulation (XGM), cross phase modulation (XPM), self phase modulation (SPM), and four waves mixing (FWM) characters, which are the main nonlinear functions in SOAs are the responsible features for developing the logic elements. In principle change of carrier density with the change of passing signal through it is principal cause for SOAs non-linearity [11].

The authors in this paper report an analytical method for conducting XOR logic operation (which is an important member of the Boolean logic family) in all-optical domain by the application of squeezed state formation of light. Squeezed states of light can be achieved here from the degenerate parametric amplification.

2. Squeezed state of light as a noise free signal:
It is well known that in squeezed states the variance of a specific squeezed quadrature becomes less than that of the vacuum or coherent state, whereas the variance of other quadrature is increased according to uncertainty relation. Thus one can use a signal of the squeezed domain in communication by the proper modulation of the amplitude of the squeezed quadrature component. A phase sensitive homodyne detector responding to the squeezed quadrature component on the receiver side can be used to detect the squeezed signal with a very high signal to noise ratio. In parametric amplification it is possible to amplify one of the two phase quadratures at an expense of extra noise in the other one. It has been well demonstrated that to achieve the long-term stability, optical parametric amplification (OPA) is much better than optical parametric oscillation (OPO). On the other hand,
the frequency of output wave from OPA can be locked by the frequency of the respective injected signal, so that the proper investigation on OPA can provide some meaningful informations to develop the tunable OPA. Therefore one can choose the parametric amplification process for the generation of squeezed state of light source as it can provide a noiseless amplifier. In quantum communication the applications such as quantum teleportation and the dense coding squeezed state of light opened the door for the framing some of complicated relations among the continuous variables because of high degree of noise reduction. For the improvement of precision measurement beyond Standard quantum limit (SQL)  in Mach-Zehnder[13] and polarization interferometry[13] quadrature squeezed state of light can also be used. The squeezed twin beams with different intensities have been used in some optical measurements in sub-shot-noise range such as in small signal recovery[14], measurements of transmissivity[15], amplitude modulated signal[16] and two-photon absorption spectroscopy[17,18]. In many practical applications the stability and reliability of these non-classical light generation systems have drawn a great importance.

3. Generation of squeezed states via Parametric Amplification:

Squeezed state of light has been generated several times in various laboratories with different mechanism. Among them degenerate parametric generation is one of the most suitable mechanism as the noise level in homodyne detection has been reduced more than 50% in comparison to the level set at the vacuum state of light[19]. The complex field expression can be written as

\[ E = E_\omega + E'_\omega \]

The output from the respective optical parametric amplifier is

\[ E^+(x) = E^+_0 e^{g \omega t} \]

and that from the concerned parametric demagnified output is

\[ E^-(x) = E^-_0 e^{-g \omega t} \]

Where \( g = ig_0 e^{\text{int}} \) with \( E_0 \), \( g \) are respectively real number, phase, and nonlinear coupling term and \( E^+ = E_+ + E_\omega \). The amplification and demagnification occurs simultaneously in \( E^+ \) and \( E^- \) respectively. The field \( E^\omega \) is proportional to \( e^{-\text{int}} \). Thus both of \( E^+ \) and \( E^- \) are directly proportional to the respective field quadrature. Hence

\[ E^- = E'_\omega - E_\omega = 2iE^X\sin \omega t \]

\[ E^+ = E'^+ \omega + E'_\omega = 2E^X\cos \omega t \]

The above expressions show that one of the field quadrature gets amplified where as the other one gets demagnified. This character exploited for the generation of a squeezed state of light[19,20].

Now the pump energy for the optical parametric amplification is supplied by the pump beam at frequency \( 2\omega \) as shown in Fig.1. This pump beam is generated by frequency doubling process with the help of non-linear crystal. Here the signal field at frequency \( \omega \) and the pump field generate an idler at frequency \( \omega \). The idler and the pump fields also generate some more signals. In this way the sub-harmonic field is produced in squeezed state.

4. The analytical approach for conducting the XOR logic operation with squeezed state of light:

It is commonly known in quantum optics that single-mode coherent squeezed states are generated through nonlinear optical interactions between coherent optical fields and nonlinear media. The two signal fields \( E_s \) and \( E'_s \) emerging from two parametric amplifiers are applied into the input port of a 50:50 beam splitter (Fig. 1). The output fields \( E_1 \) and \( E'_2 \) from the beam splitter can be expressed as

\[ E_1 = 1/\sqrt{2}(E_s e^{i\varphi} + E'_s) \]  
\[ E_2 = 1/\sqrt{2}(E_s e^{i\varphi} - E'_s) \]

Where \( \varphi \) is the phase of \( E_s \) signal relative to \( E'_s \) signal. A 50:50 beam splitter always introduce a relative phase shift of \( \pi \) between the two output ports. Hence negative sign arises in Eq (1.2). Using signal fields in complex domain one can write

\[ E_s = E^{X_1} + iE^{X_2} \]  
\[ E'_s = E^{X_1} + iE^{X'_2} \]

Then in complex time domain

\[ E_1 = 1/\sqrt{2}[(E^{X_1}_s + iE^{X'_2}_s)(\cos \varphi + i\sin \varphi)] + (E^{X_1}_s + iE^{X'_2}_s)] \]

\[ = 1/\sqrt{2}[(E^{X_1}_s \cos \varphi - E^{X'_2}_s \sin \varphi + E^{X'_1}_s) + i(\overline{E^{X_1}_s \sin \varphi + E^{X'_2}_s \cos \varphi + E^{X'_1}_s})] \]

\[ E_2 = 1/\sqrt{2}[(E^{X_1}_s + iE^{X'_2}_s)(\cos \varphi + i\sin \varphi)] - (E^{X_1}_s + iE^{X'_2}_s)] \]

\[ = 1/\sqrt{2}[(E^{X_1}_s \cos \varphi - E^{X'_2}_s \sin \varphi - E^{X'_1}_s) + i(\overline{E^{X_1}_s \sin \varphi + E^{X'_2}_s \cos \varphi - E^{X'_1}_s})] \]

So the subtracted output \( (O_p) \) of the system shown in Fig.1

\[ O_p \propto i_1 - i_2 \]

\[ \propto (E_1E'_1 - E_2E'_2) \]

Now we evaluate \( E_1E'_1 - E_2E'_2 \) with eqn (3.1), eqn (3.2) and taking their conjugates which yields
\[ E_1 E_2 = 1/2 [(E_X^1 \cos \phi - E_X^2 \sin \phi + E_Y^1)^2 - i^2 (E_X^1 \sin \phi + E_X^2 \cos \phi + E_Y^2)] \]

\[ = 1/2 [(E_X^1 \cos \phi - E_X^2 \sin \phi + E_Y^1)^2 + (E_X^1 \sin \phi + E_X^2 \cos \phi - E_Y^2)^2] \quad (5.1) \]

and

\[ E_2 E_2 = 1/2 [(E_X^1 \cos \phi - E_X^2 \sin \phi - E_Y^1)^2 - i^2 (E_X^1 \sin \phi + E_X^2 \cos \phi - E_Y^2)] \]

\[ = 1/2 [(E_X^1 \cos \phi - E_X^2 \sin \phi - E_Y^1)^2 + (E_X^1 \sin \phi + E_X^2 \cos \phi - E_Y^2)^2] \quad (5.2) \]

\[ ∴ E_1 E_1 - E_2 E_2 = 1/2 [(E_X^1 \cos \phi - E_X^2 \sin \phi + E_Y^1)^2 - i^2 (E_X^1 \sin \phi + E_X^2 \cos \phi + E_Y^2)] -
\]

\[ [(E_X^1 \cos \phi - E_X^2 \sin \phi - E_Y^1)^2 - (E_X^1 \sin \phi + E_X^2 \cos \phi - E_Y^2)^2] = 2(E_X^1 E_Y^1 \cos \phi +
\]

\[ E_X^2 E_Y^1 \sin \phi + E_X^1 E_Y^2 \sin \phi - E_X^2 E_Y^1 \sin \phi) \]

\[ = 2(E_X^1 E_Y^1 + E_X^2 E_Y^2) \cos \phi - (E_X^1 E_Y^1 - E_X^2 E_Y^2) \sin \phi \]

\[ (6) \]

To obtain zero output i.e. no field we must have

\[ E_1 E_1 - E_2 E_2 = 0 \]

\[ \tan \phi = (E_X^1 E_Y^1 + E_X^2 E_Y^2) / (E_X^1 E_Y^1 - E_X^2 E_Y^2) \]

\[ (7) \]

From the definition of squeezed signal given in eqn(2.1) and (2.2) one can write their components as given below

\[ E_X^1 = E_0 \cos (\omega t - kx + \phi) \]

\[ E_X^2 = E_0 \sin (\omega t - kx + \phi) \]

\[ E_Y^1 = E'_0 \cos (\omega t - kx) \]

\[ E_Y^2 = E'_0 \sin (\omega t - kx) \]

\[ \tan \phi =
\]

\[ = \frac{E_0 \cos (\omega t - kx + \phi) \cos (\omega t - kx) \sin (\omega t - kx + \phi) \sin (\omega t - kx)}}{E_0 \cos (\omega t - kx + \phi) \cos (\omega t - kx) - \sin (\omega t - kx + \phi) \sin (\omega t - kx)}} \]

\[ \Rightarrow 2\phi = \frac{\pi}{2} \]

\[ \Rightarrow \phi = \frac{\pi}{4} \]

\[ (9) \]

So this condition forces the output \( O_p = 0 \) in presence of two squeezed state of lights.

5. Operation with necessary optical circuit:

The light from the laser source A divides into two parts upon falling on the beam splitter (B,M) as shown in fig.1. One part of the light goes through \( \chi^{(2)} \) medium which doubles the frequency and the other goes to Optical Parametric Amplifier (OPA) through mirror. The intense field at frequency \( 2\omega \) acts as pump and that of weak field at \( \omega \) acts as signal. In OPA the pump and signal give birth to idler at \( 2\omega - \omega = \omega \) in non-linear crystal by difference frequency generation. After mixing the pump and the idler waves in the non-linear crystal it produces more signal at \( \omega \) and so on. In this way the signal field grows. The signal field gets instantaneous amplification or de-amplification depending on its phase relative to the pump field. This is the principle of generation of a squeezed state signal field. The same process of generation squeezed state of signal field repeats by the laser B also. Here the phase shifter introduces a relative phase difference of \( \pi/4 \) between the two signal fields. These two squeezed signal fields generated from Laser A and B with a relative phase difference of \( \pi/4 \) are then fall into a 50:50 beam splitter. The outputs from the beam splitter are connected to the two photodiodes (D,D) for necessary homodyne detection to get the final logical output at O/P.

![Fig1.: Schematic diagram for the realization of X-OR operation.](image)

| Table 1: Truth Table for X-OR Logic Gate |
|-----------------|-----------------|-----------------|
| Input Signal Field (A) | Input Signal Field (B) | Output Signal Field (O/P) |
| No Field (0) | No Field (0) | No Field (0) |
| Squeezed Field(1) | No Field (0) | Squeezed Field(1) |
| No Field (0) | Squeezed Field(1) | Squeezed Field(1) |
| Squeezed Field(1) | Squeezed Field(1) | No Field (0) |
6. Conclusion:
We have taken this operation for implementation with squeezed state of light as X-OR logic operation has a tremendous application in digital processing. The squeezed signal is found when only anyone signal at the input side exists, otherwise no light is found. Therefore at the output side a signal of squeezed state indicates 1 and that of a non-squeezed state or no light indicates 0 to implement the logic operation. X-OR logics are used in almost all the data comparisons very successfully, therefore this mechanism can be used for optical data comparisons and for sending a compared data at long distance. One can implement also the other members of the logic family adopting the interaction among squeezed state of light. The optics inversion operation (NOT Logic) is also executed using the technique. The whole system is all-optical and the parallelism of optics are exploited here to get the superfast speed of operation.

References: