Velocity Changing and Dephasing collisions Effect on electromagnetically induced transparency in V-type Three level Atomic System.

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Abstract: We study electromagnetically induced transparency in an inhomogeneously broadened three level V-type atomic system including residual Doppler broadening which arises due to pump-probe wave vector mismatch. We show that the velocity changing collisions (VCC) and dephasing collisions (DPC) play a key role in creating transparency at exact resonance at relatively low coupling field intensities.

1. INTRODUCTION

Electromagnetically induced transparency [EIT][1] is an effect observed when a weak probe beam propagating through medium, on resonance with an atomic transition shows reduced absorption due to the presence of a strong coupling beam on another linked transition. In addition to reduced absorption the modification of dispersion properties of the medium leads to subluminal or superluminal group velocity of the transmitted probe. Accordingly EIT has received considerable attention in over the years as it offers a variety of interesting and potentially important applications including light storage [2-4], quantum information [5] and precision magnetometers [6,7].

Study and identification of various mechanisms that lead to broadening of EIT resonance has attracted considerable attention in recent years. Three level models of various types of ladder, V and Λ configurations are most commonly employed for studying EIT in vapors or solids. The experimental observation of cw wave vector mismatched transparency in a V-type Doppler broadened system has been reported by several authors earlier [8-10].

In this work we present a general theory in an inhomogeneously broaden V-type atomic system in which Doppler broadening of both one-and two photon transitions occurs. The V-type three level systems involved in the present study of EIT is shown in Fig.1. The theory is more general in the sense that it takes into account the residual Doppler broadening of the two photon coherence in different wave-vector mismatch regimes and also the collisional effects of an additional (buffer) gas in vapor cell. Both the velocity changing and dephasing aspects of collisions with atoms of the buffer gas are considered. The numerical calculations are based on a simple but often applicable collision model which facilitates inclusion of the effect of both the velocity-changing collisions and dephasing collisions on level coherence. In Sec.2, a theoretical formalism is presented, where we solve for the steady-state density-matrix equations for this system and perform a Doppler averaging over the atomic susceptibility and expressions for probe absorption coefficient are obtained. Numerical results for probe absorption profiles are presented in Sec.3.

![Fig 1: The three level V-system. Ω_p and Ω_c respectively are the coupling and probe field Rabi frequencies applied to transitions 1↔2 and 1↔3.](image-url)
spontaneous emission rate from the upper level $|3>$ to intermediate level $|2>$ is $S_{32}$ and that from level $|2>$ to ground level $|1>$ is $S_{21}$. A coupling field $E_\epsilon (=-\epsilon_{\epsilon}\exp[i(\vec{k}_\epsilon \cdot \vec{r} - \omega_\epsilon t) + \text{c.c.}]$ of frequency $\omega_\epsilon$ wave vector $\vec{k}_\epsilon$ and Rabi frequency $\Omega_\epsilon = (\vec{\mu}_\epsilon \cdot \vec{E}_\epsilon) / \hbar$ is driving the $|1>\leftrightarrow |2>$ transition and a weak probe field, $E_p (=-\epsilon_p\exp[i(\vec{k}_p \cdot \vec{r} - \omega_p t) + \text{c.c.}]$ of frequency $\omega_p$ wavevector $\vec{k}_p$ and Rabi frequency $\Omega_p = (\vec{\mu}_p \cdot \vec{E}_p) / \hbar$, is applied to the $|1>\leftrightarrow |3>$ transition. Here $\vec{\mu}_{12}$ and $\vec{\mu}_{13}$ are the dipole moment of $|1>\leftrightarrow |2>$ and $|1>\leftrightarrow |3>$ transition respectively. The interaction Hamiltonian $V^{\text{int}}$ under near resonant conditions and rotating wave approximation is

\[
V^\text{int} = \hbar [\Omega_c \exp[i(\vec{k}_c \cdot \vec{r} + \Delta c t)\hat{\sigma} + \text{c.c.}]] + \frac{\Omega_p \exp[i(\vec{k}_p \cdot \vec{r} + \Delta p t)]\hat{\sigma} + \text{c.c.}}{\hbar}]
\]

(1)

Where $\Delta_c = (\omega_{21} - \omega_c)$ and $\Delta_p = (\omega_{31} - \omega_p)$ denote detuning of the probe and control field frequencies from atomic resonance frequencies $\omega_{21}$ and $\omega_{31}$ respectively and $\hat{\sigma} = (i, j = 1, 2, 3)$ are the atomic raising or lowering operators.

**Density matrix- Equation of motion:**

We use the density matrix formalism as it allows us to incorporate the effect of various decay mechanisms on populations of the atomic levels involved and the coherences established between them. It is possible to relate atomic susceptibilities to these coherences and thus inspect the absorption and dispersion of the light incident on the atom. The time evolution of the density matrix of the system in the interaction picture is

\[
\dot{\rho}_m = \sum_{j,k} V^\text{int}_{mn} V^\text{int}_{jk} \rho_{nk} + (\dot{\rho}_m)_\text{Rel}
\]

(2)

where $\rho_m = \langle j | \hat{\rho} | k \rangle$ and $V^\text{int}_{mn} = \langle m | V^\text{int} | n \rangle$. ($j, k, m, n = 1, 2, 3$) can be calculated using the interaction Hamiltonian $V^\text{int}$ given by Eq.(1). Doppler shift of atomic resonance due to thermal motion of atoms in the medium gives rise to inhomogeneous broadening. To incorporate atomic motion, the derivative $\dot{\rho}_m$ on the left hand side of Eq.(2) can be replaced by

\[
\dot{\rho}_m \rightarrow \{(\partial / \partial t) + \vec{v} \cdot \nabla\} \rho_m + \vec{v} \cdot \nabla\rho_m
\]

(3)

where $\vec{v}$ is the atomic velocity. For brevity we shall henceforth denote a function $f(v_s, v_r, v_c)$ by $f(v)$. The second term in Eq.(2) describes phenomenological inclusion of the effect of relaxation processes such as spontaneous emission $S_{jk}$ ($j,k=1,2,3$), radiative decay of off-diagonal elements $\gamma_{jk} (= S_{jk}/2$), and collisions in the system. Collisions with buffer gas atoms can cause dephasing as well result in changes in the velocity of a vapor atom. Effect of such collisions can be incorporated in the density matrix formalism by including a term $[24]

\[
\dot{\rho}_m = -\gamma_{ph}(1 - \delta_{ph}) \rho_m (v,t) - \Gamma_{jk} \rho_{jk} (v,t) + \int W_{jk} (v' \rightarrow v) \rho_{jk} (v',t) \text{d} v'
\]

(4)

Here $\gamma_{ph}$ is the rate of collision induced dephasing of optical coherences and $\Gamma_{jk}$ is some average rate of change in velocity $v$. For simplicity $\Gamma_{jk}$ is assumed to be independent of $v$ and related to the collision kernel $W_{jk} (v' \rightarrow v)$ by

\[
\Gamma_{jk} = \int W_{jk} (v' \rightarrow v) \text{d} v'
\]

(5)

The collision kernel in general is assumed to be of the form

\[
W_{jk} (v' \rightarrow v) = W_j (v - \alpha v'),
\]

(6)

Where $\alpha$ is a constant, ($1 > \alpha > 0$). Physically the second term in Eq.(2,5) is the ‘out term’ representing collisional shift of atoms with a velocity $v$ to other velocity subclasses at some rate $\Gamma_{jk}$ and the third term is the ‘in term’ arising due to collisional shift of atoms from other velocity subclasses into velocity subclass $v$. In this work we however restrict our discussion to the case of most prevalent and experimentally relevant strong collision case in which collisions result in rapid thermalization of the velocity distribution of the system.

**Strong collision model**

In this case a single collision on average, thermalizes the velocity shift distribution regardless of the initial velocity i.e., the collision kernel of Eq.(2,7) is assumed to independent of initial velocity
Our aim is to determine the velocity averaged first order photon coherence \( \Gamma^{(1)} \) of the form
\[
\Gamma^{(1)} = \int \rho^{(1)}(v,t) \, dv
\]
where
\[
\rho^{(1)}(v,t) = \rho_{13}^{(1)}(v,t) + \rho_{12}^{(1)}(v,t)
\]
and of the form
\[
W_{\nu}(v' \rightarrow v) = \Gamma_{\nu} M(v),
\]
where
\[
M(v) = \left( v \sqrt{\pi} \right)^{-i} \exp[-v \cdot \tilde{v} / v_0^2] \]
is the Maxwellian velocity distribution and \( v_0 = (2k_B T / m)^{1/2} \) is the most probable thermal velocity at a temperature \( T \) of an atom of mass \( m \).

The equations describing time evolution of the slowly varying components of the density matrix elements \( \tilde{\rho}_{jk}(v,t) \), can now be written using Eq.(1),(3)-(4) and (7) in Eq. (2) and appropriate transformations to eliminate fast oscillating terms .

\[
\dot{\rho}_{13} = i \Omega_{13} \rho_{13} - \Omega_{23} \rho_{23} - (\gamma_1 + \gamma_3) \rho_{13} + i \left( \Delta_p - \Delta_c \right) \rho_{13}
\]
\[
+ k_p (v_0) \rho_{13}
\]
\[
\dot{\rho}_{12} = i \Omega_{12} \rho_{12} - \Omega_{22} \rho_{22} - (\gamma_1 + \gamma_2) \rho_{12} + i \left( \Delta_p - \Delta_c \right) \rho_{12}
\]
\[
+ k_p (v_0) \rho_{12}
\]
\[
\dot{\rho}_{23} = i \Omega_{23} \rho_{23} - \Omega_{13} \rho_{13} - (\gamma_2 + \gamma_3) \rho_{23} + i \left( \Delta_p - \Delta_c \right) \rho_{23}
\]
\[
+ k_p (v_0) \rho_{23}
\]
\[
\dot{\rho}_{22} = i \Omega_{22} \rho_{22} - \Omega_{12} \rho_{12} - (\gamma_2 + \gamma_1) \rho_{22} + i \left( \Delta_p - \Delta_c \right) \rho_{22}
\]
\[
+ k_p (v_0) \rho_{22}
\]
\[
\dot{\rho}_{33} = i \Omega_{33} \rho_{33} - \Omega_{13} \rho_{13} - \gamma_3 \rho_{33} - i \left( \Delta_p - \Delta_c \right) \rho_{33}
\]

Our aim is to determine the velocity averaged first order photon coherence \( \Gamma^{(1)} = \int \rho^{(1)}(v,t) \, dv \) the imaginary and real part of which describes probe absorption and dispersion, respectively, in the three level V-type system.

Under Steady state conditions the density matrix equation is to be solved. Initially all the population is in the level \( |2> \rightarrow |1> \) by the application of strong control field. The zeroth order solutions are obtained under the assumption \( \rho^{(0)}_{13} + \rho^{(0)}_{23} = M(v) \) The Zeroth and first order matrix elements are
\[
\rho^{(0)}_{11}(v,t), \rho^{(1)}_{22}(v,t), \rho^{(1)}_{13}(v,t), \rho^{(1)}_{23}(v,t).
\]
The relevant first order density matrix equations are found as
\[
\dot{\rho}_{23}^{(1)} = -i(\Delta_p - \Delta_c) + (k_p - k_c) \nu_x + \gamma_1 + \gamma_2 + \gamma_3 + \nu_p \rho^{(1)}_{23}
\]
\[
i \Omega_{23} \rho^{(2)}_{23} - \Omega_{13} \rho^{(2)}_{13} - \gamma_2 \rho^{(2)}_{23} + \gamma_1 \rho^{(2)}_{13} + \Omega_{23} M(v) \int \rho^{(1)}_{23}(v,t) \, dv
\]
\[
\dot{\rho}_{33}^{(1)} = -i(\Delta_p + k_p \nu_x) + \gamma_3 + \gamma_1 + \nu_p \rho^{(1)}_{33}
\]
\[
i \Omega_{33} \rho^{(2)}_{33} = -i \left( \gamma_3 + \Delta_c + k_c \nu_x \right) \rho^{(2)}_{33} + \Omega_{33} M(v) \int \rho^{(1)}_{33}(v,t) \, dv
\]
the study state solution obtained by setting the time derivative to zero on the left-hand side of Eq.(11) yields the velocity averaged one-photon coherence as
\[
\int \rho^{(1)}_{ij}(v) \, dv = -i \Omega_{ij} \int \frac{(-A_{ij} \rho^{(0)}_{ij} - \Omega \rho^{(1)}_{ij}) \, dv + i \gamma_1 A_{ij} \rho^{(2)}_{ij} + \Omega_{ij} M(v) \int \rho^{(1)}_{ij}(v,t) \, dv}{A_{ij} A_{ij} + \Omega^2}
\]
Where \( A_{ij}, A_{ij} \) and \( \Gamma^{(1)}_{ij} \) are given below
\[
A_{ij} = (i(\Delta_p - \Delta_c) + (k_p - k_c) \nu_x + \gamma_1 + \gamma_2 + \gamma_3 + \nu_p)
\]
\[
\Gamma^{(1)}_{ij} = i \Omega_{ij} \int \frac{(- \Omega \rho^{(0)}_{ij} - \Omega \rho^{(1)}_{ij}) \, dv + i \gamma_1 A_{ij} \rho^{(2)}_{ij} + \Omega_{ij} M(v) \int \rho^{(1)}_{ij}(v,t) \, dv}{A_{ij} A_{ij} + \Omega^2}
\]

3. Numerical result and discussions:
We now present numerical results for EIT applying the theory to an atomic vapor contained in a cell in which a buffer gas is also introduced. The velocity changing collision rate \( \Gamma_{23} \) (i.e. dephasing rate \( \gamma_p \) ) can be varied by changing the pressure of the buffer gas in the vapor cell. It was found in many earlier studies [11, 12] that the nature of EIT in a three level system depends critically on the sign of the residual Doppler width \( \delta k = (k_p - k_c) \) which depending upon the probe and control field wavevector mismatch, is either positive \( (k_p > k_c) \) or negative \( (k_p < k_c) \). For very large wave vector mismatch it was shown in Ref. 12 that EIT in a V system is markedly dissimilar in these two cases. In our study in the wave vector mismatch in \( ^8 \text{Rb} \) \( D_1 \) and \( D_2 \) transitions is very small compared with that in earlier studies [11, 12].

In Fig. 2 (i) the absorption profile is shown for...
positive mismatch \((k_p > k_c)\) regime of residual Doppler broadening. The absorption profile shown in Fig 2(ii) is for a negative mismatch \((k_p < k_c)\) regime of residual Doppler broadening.

![Absorption Profile](image)

The appearance of a transparency dip at exact resonance shows that with increase in VCC rates at fixed dephasing and coupling field intensities absorption is reduced at exact resonance. The transparency is much more in the case of negative mismatch \((k_p < k_c)\) than that compared with the positive mismatch \((k_p > k_c)\) regime. Therefore the residual Doppler broadening arising due to pump-probe wave vector mismatch is suppressed by VCC which also enhances transparency in both wave vector regimes.

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### 4. REFERENCES
