NUMERICAL ANALYSIS OF STABILITY OF FIBER MODE LOCKED LASER

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Abstract: Stability of an Er$^{3+}$ doped actively mode locked fiber laser is derived using the master equation. The loop stability under polarization mismatch and modulation frequency detuning is analyzed. The variation in harmonic power is considered for defining the stability criteria. Jones matrices have been used to model fiber optic components. Comparison of experimental data with simulated results is also presented.

1. INTRODUCTION

Mode locking is a phenomenon in which the cavity resonant frequencies of the laser are forced to have matched phase shifts. This effect causes the waves to constructively interfere to form ultra-short pulses. This effect has been analyzed by various researchers in the cavities containing He-Ne as well as Nd:YAG lasers [1]. Here we present a numerical analysis of an all fiber ring mode locked laser. We intend to demonstrate the effect of polarization mode dispersion and modulation frequency shifts away from centre frequency on mode-locking stability.

2. DESCRIPTION OF MLL AND MODELING

Harmonically FM Mode locked laser in our all fiber case consists of Er$^{3+}$ doped gain medium, WDM couplers, Power dividers, Polarizer, LiNbO$_2$ modulator. The MLL is made of normal SMF components. The schematic of the MLL is as Fig 1.

2.1 Master equation model

The mode locked loop is modeled using a so called master equation for mode locking [1]. This equation can be used to understand the behavior of the mode locked loop with considerable accuracy. The master equation is written as

$$T_r \frac{dA}{dt} = \left[-l + \sum_{n=2}^{m} \frac{D_n (j \frac{\partial}{\partial t})^n}{\Delta \omega_m t} + g \left(1 + \frac{1}{2} \frac{\partial^2}{\partial t^2} \right) - M (1 - \cos(\Delta \omega_m t)) \right] A$$

(1)

Where $T_r$, $g$, $D_n$, $M$, $\Delta \omega_m$ are round trip time, gain of EDFA, gain dispersion, $n$th order dispersion, modulation index and modulation frequency respectively. This equation can be solved using various techniques like Split Step Fourier Transform, finite difference methods etc. Our approach is to use spectral methods in Fourier Galerkin construct [2]. This can be done by assuming the solution to be of the form

$$A(T, t) = \sum_{k=-N/2}^{N/2} a_k(T)e^{-jk\omega_m t}$$

(2)

We can convert the PDE in (1) to a system of differential equations which can be represented by a matrix as

$$\frac{da_k}{dt} = T a_k$$

(3)

and

$$T_{ii} = \frac{\gamma_{N/2}}{T_r}, T_{i,i+p} = T_{i,i-p} = \frac{M}{T_r}$$

(4)

$$\gamma_{N/2} = (g - l - M) + j \sum_{n=2}^\infty (j \omega_n k)^n D_n$$

(5)

We can convert this to

$$\Delta D = -\frac{T_r^2}{2\pi} \Delta \omega_m$$

(6)

2.2 Including Detuning Effects

Since (1) is in retarded frame of reference there is no group velocity term. In order see the effect of detuning we introduce small perturbation of group velocity as $j \omega_0 k \Delta D$ in (5). $\Delta D$ and $\Delta \omega_m$ can be related as

$$\Delta D = -\frac{T_r^2}{2\pi} \Delta \omega_m$$

(7)

2.3 Including Polarization mode dispersion effects

To model PMD we use an approach which is similar to that used in literature for long distance networks [3][4]. Commonly modeling of the PMD is done for long haul communication networks. Modeling of PMD for small loop devices such as MLL is also essential because in these devices a pulse will be circulating in the cavity which in effect is similar to a pulse propagating in the long distance fibers. The PMD effects hence accumulate and places restriction on minimum PW that can be generated. There are two aspects of PMD, one being the polarization axis rotation because of environmental fluctuations and manufacturing induced random birefringence along the fiber length and the group delay variation in fast axis and slow axis of the fibers. The polarization axis rotation causes the coupling of the two modes of the fiber and the differential group delay would cause the pulses in the two axis detach from each other, thereby increasing the pulse width.

In order to include PMD in the Master mode lock equation we need vector form (3) written as (7).
\[
\frac{d}{dt} \begin{bmatrix} a_k^x \\ a_k^y \end{bmatrix} = R^{-1} \begin{bmatrix} T_x & 0 \\ 0 & T_y \end{bmatrix} R \begin{bmatrix} a_k^x \\ a_k^y \end{bmatrix}
\]

Assuming that all the frequency components undergo same amount of polarization axis rotation we can write

\[
R = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix}
\]

\(\alpha\) is the random axis rotation angle.

The origin of polarization axis rotation in this MLL configuration can be identified as the points where two components are being connected, for example at the input and output of the individual components. Since fibers attached to these components are not PM fibers, random polarization rotations can occur at these interfaces.

To numerically model this behavior we have identified 10 points as indicated in Fig 1 are points of PMD. In the time duration of single \(T_r\) 10 polarization rotations are introduced at equal intervals. The rotation angle is a uniform distribution between 0 and 2\(\pi\). The random group delay variation is Gaussian distributed with variance equal to the value of DGD specified by the fiber manufacturer.

The effect of modulator polarization is also critical in estimating the PMD effects. In all fiber mode locked loops, the typical LiNbO\(_3\) modulators used are polarization sensitive. They have high extinction ratio between the modulations of two polarizations.

3. NUMERICAL SIMULATION

In the general behavior of the mode locked loop is the reduction of pulse width per round trip, this occurs by generation of frequency components at \(2\pi n\omega_m\) radians/sec in every round trip. Even though \(2\pi n\omega_m\) components are generated at each round trip the power associated with the peripheral frequency components is very low after a large number of round trips and depends on the modulation index. So a large number of frequency points would lead to slowing down of the simulation at initial round trips when actual number of frequency components required is low. Smaller number of components will degrade the accuracy of the simulation at larger round trip numbers. In order to compromise between accuracy and time taken for simulation we need an adaptive approach for number of frequency points to be used in simulation. Adaptive approach was successfully used to simulate this system represented by (7) for various conditions and results are presented.

4. RESULTS

4.1 Simulation Results

Simulation was performed to study the behavior of the cavity under various conditions such as frequency detuning and effect of Polarization Mode Dispersion on the pulse width. Fig 2 shows the variation of FWHM with respect to the frequency detuning of the cavity at a modulation frequency of 10MHz. We see that there is a finite frequency range beyond the optimum modulation frequency where the pulse width is not changing. This behavior is primarily due to the low modulation frequency used. At these low frequencies there is higher coherence between the waves, which results in insensitivity to frequency detuning. The results shown are for simulations run over 1000 round trips. We have noticed that higher number of round trips leads only to small pulse width reduction per round trip. This only re-forces the fact that we need to use high modulation frequencies (as used in harmonic mode-locking) for generation of ultra short pulses.

Stabilizing the loop with respect to environmental factors is another matter of concern. This is due to the fact that detecting pulse width of short pulses in order to correct instabilities in the system is extremely difficult in real time. Another method to correct for stability is to use the harmonics of the pulse as detected by the photo diode. Fig 3 shows the plot of 6 harmonics of the pulse, which can be used as bench mark for detecting inaccuracies. As seen from the graph, monitoring higher harmonics will be a more accurate indication of instabilities in the system.

The effect of PMD is very minimal at large pulse widths generated by 10MHz modulation frequency. In order to investigate the role of PMD in ultra-short (< 1 ps) pulse lasers, simulation was done at high frequency of 10GHz with a modulator extinction ratio of 10dB (here we assume that x-polarized wave is preferred by the modulator). The results for such a case with and without PMD effect (DGD = 9ps/\(\sqrt{\text{km}}\)) is illustrated in Fig 4. As seen in Fig 4, we can clearly see the cross coupling between two polarizations for ultra short pulses. The y-polarized wave manifests itself as a short pulse adjoining the x-polarized pulse, thereby increasing the overall pulse width. Finally, the PMD causes random perturbations in the pulse width as in Fig 5, which needs to be controlled for steady operation. An histogram that further illustrates this behavior is in Fig 6.

4.2 Experimental results

A mode locked loop was built to test the performance of practical MLL and the detuning performance is presented in Fig 2. The loop length used was about 20m and Erbium doped fiber was used as gain medium. Comparing experimental and practical data from Fig 2 we see that model underestimates the detuning effects. We are currently investigating this issue and will discuss the same at the Conference.

Variation of harmonics with frequency detuning is presented in Fig 7.
5. Conclusion

In this paper we have used the spectral methods for solving the master mode locking equation. Modifications are done on the basic equation to include the effects of frequency detuning and polarization mode dispersion effects. Finally we have also compared the results of the solution with experimental data.

5. Figures

1. Block Diagram of MLL

2. Pulse width Vs Frequency detuning

3. Variation of Harmonics with Frequency detuning

4. Pulse without/with PMD after 252 round trips

5. Pulse width variation in the presence of PMD

6. Histogram of PW variation due to PMD

7. Experimentaly measured results of harmonics

REFERENCES